FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2019
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

PURE MATHEMATICS

Roll Number

TIME ALLOWED: THREE HOURS	**MAXIMUM MARKS = 100**

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Show that the order and the index of a subgroup divides the order of a finite group. (10)

(b) Show that every finite integral domain is a field. (10) **(20)**

Q. 2. **(a)** Show that the characteristic of an integral domain is R is either zero or a prime. (10)

(b) Determine whether or not the set $\{(1, 2, -1), (0, 3, 1), (1, -5, 3)\}$ of vectors is a basis for R^3. (10) **(20)**

Q. 3. **(a)** Show that a one-to-one linear transformation preserves basis and dimension. (10)

(b) Solve the system of linear equations: (10) **(20)**

$$2x_1 + x_2 + 5x_3 = 4$$
$$3x_1 - 2x_2 + 2x_3 = 2$$
$$5x_1 - 8x_2 + 2x_3 = 1.$$

SECTION-B

Q. 4. **(a)** Solve $\int_0^{\frac{\pi}{2}} \sin^2 6x \cos^4 3x \, dx.$ (10)

(b) Find the area enclosed by $y = \frac{6}{2-\cos\theta}$. (10) **(20)**

Q. 5. **(a)** Show that in any conic semi-latusrectum is the harmonic mean between the segments of focal chord. (10)

(b) Prove that the evolute of hyperbola (10) **(20)**

$$2xy = a \text{ is } (x+y)^{\frac{2}{3}} - (x-y)^{\frac{2}{3}} = 2a^{\frac{2}{3}}.$$

Q. 6. **(a)** Define Supremum and Infimum of a sequence. Find the supremum and infimum of the set (10)

$$\left\{(-1)^n \left(1 - \frac{1}{n}\right), n = 1,2,3 \ldots\right\}.$$

(b) Evaluate (10) **(20)**

$$\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} - e}{x}.$$

SECTION-C

Q. 7. **(a)** Show that $Log(1 + \cos\theta + i\sin\theta) = \ln(2\cos\frac{\theta}{2}) + i\frac{\theta}{2}$. (10)

(b) Find v such that $f(z) = u + iv$ is analytic. (10) **(20)**

Q. 8. **(a)** Prove that the series $z(1-z) + z^2(1-z) + z^3(1-z) + \cdots$ converges for $|z| < 1$, and find its sum. (10)

(b) Find the residues of $f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}$ at all its poles in the finite plane. (10) **(20)**

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