FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2016
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

PURE MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: (i) Attempt **FIVE** questions in all by selecting **TWO** Questions each from **SECTION-A&B** and **ONE** Question from **SECTION-C. ALL** questions carry **EQUAL** marks.
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed.**

SECTION-A

Q. 1. **(a)** Prove that the normaliser of a subset of a group G is a Subgroup of G. (10)

(b) Let A be a normal subgroup and B a subgroup of a group G. Then prove that $< A,B > = AB$ (10) **(20)**

Q. 2. **(a)** Let a be a fixed point of a group G and consider the mapping $I_a : G \to G$ defined by $I_a(g) = aga^{-1}$ where $g \in G$. (10)

Show that I_a is an automorphism of G. Also show that for $a, b \in G$, $I_a . I_b = I_{ab}$ (10) **(20)**

(b) Let $M_2(R) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in R \right\}$ be the set of all 2×2 matrices with real entries. Show that $(M_2(R), +, \cdot)$ forms a ring with identity. Is $(M_2(R), +, \cdot)$ a field?

Q. 3. **(a)** Let $T: X \to Y$ be a linear transformation from a vector space X into a Vector space Y. Prove that Kernal of T is a subspace. (10)

(b) Find the value of λ such that the system of equations (10) **(20)**

$$x + \lambda y + 3z = 0$$
$$4x + 3y + \lambda z = 0$$
$$2x + y + 2z = 0$$

has non-trivial solution.

SECTION-B

Q. 4. **(a)** Using $\delta - \epsilon$ definition of continuity, prove that the function $Sin^2 x$ is continuous for all $x \in R$. (10)

(b) Find the asymptotes of the curve $(x^2-y^2)(x+2y) + 5(x^2+y^2) + x+y = 0$ (10) **(20)**

Q. 5. **(a)** Prove that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{1/e}$. (10)

PURE MATHEMATICS

Q. 6. **(a)** Find the area enclosed between the curves $y=x^3$ and $y=x$. (10)

(b) A plane passes through a fixed point (a, b, c) and cuts the coordinate axes in A, B, C. Find the locus of the centre of the sphere OABC for different positions of the plane, O is the origin. (10) **(20)**

SECTION-C

Q. 7. **(a)** Determine $P(z)$ where (10)

$$P(z) = (z-z_1)(z-z_2)(z-z_3)(z-z_4) \text{ with } z_1 = e^{i\pi/4}, z_2 = \bar{z}_1, z_3 = -z_1 \text{ and } z_4 = -\bar{z}_1.$$

(b) Find value of the integral $\int_c (z - z_0)^n \, dz$, (n any integer) along the circle C ....with centre and z_0 radius r, described in the counter clock wise direction. (10) **(20)**

Q. 8. **(a)** Use Cauchy Integral Formula to evaluate $\int_c \frac{\cosh z + \sin 2z}{z - i/2} \, dz$ along the simple closed counter C: $|z|=3$ described in the positive direction. (10)

(b) State and prove Cauchy Residue Theorem. (10) **(20)**

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