FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2021
FOR RECRUITMENTTO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE: **(i)** Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) **Use of Calculator is allowed**.

Q. No. 1. **(a)** Evaluate the surface integral $\iint \vec{A}.\vec{n}dS$ where $\vec{A} = z\vec{i} + x\vec{j} - 3y^2 z\vec{k}$ and S is the portion of the cylinder $x^2 + y^2 = 8$ lying in the first octant between $z = 0$ and $z = 4$. **(10)**

(b) Prove that **(10)**

$$\nabla(f(r)) = \frac{f^{F}(r)}{r}\vec{r},$$

where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$.

Q. No. 2. **(a)** The greatest resultant that two forces can have is of magnitude P and the least is of magnitude Q.Show that, when they act at an angle α, their resultant is of magnitude $\sqrt{P^2 cos^2\frac{\alpha}{2} + Q^2 sin^2\frac{\alpha}{2}}$ **(10)**

(b) A sphere of weight W and radius a is suspended by a string of length l from a point P and a weight w is also suspended from P by a string sufficiently long for the weight to hang below the sphere. Show that the inclination of the first string to the vertical is **(10)**

$$\sin^{-1}\frac{wa}{(W + w)(a + l)}.$$

Q. No. 3. **(a)** Show that the law of force towards the pole, of a particle describing the curve $r^n = a^n \cos n\theta$ is given by **(10)**

$$f = \frac{(n + 1)h^2 a^{2n}}{r^{2n+3}}.$$

(b) The maximum velocity that a particle executing simple harmonic motion of amplitude a attains, is v. If it is disturbed in such a way that its maximum velocity becomes nv. Find the change in the amplitude and the time-period of motion. **(10)**

Q. No.4. **(a)** Define ordinary and singular points of the differential equation $a_2(x)y'' + a_1(x)y' + a_0(x)y = 0$. When a singular point is said to be regular and irregular? Find regular and irregular singular points of the differential equation $(x^2 - 4)^2 y'' + (x - 2)y' + y = 0$. **(10)**

(b) Show that **(10)**

$$J_{3/2} = \sqrt{\frac{2}{\pi x}}\left[\frac{\sin x}{x} - \cos x\right].$$

Q. No. 5. **(a)** Solve the equation by using method of undetermined coefficients **(10)**

$$y'' - y' + y = 2\cos 3x.$$

(b) Use the method of Frobenius to find two linear independent series solutions in powers of x of the DE. **(10)**

$$x^2 y'' - (x^2 + x)y' + y = 0.$$

Q. No. 6. **(a)** Classify general second order partial differential equation (PDE) into elliptic, parabolic and hyperbolic form. Discuss the nature of the PDE **(10)**

$(1 - x^2)u_{xx} - 2xyu_{xy} + (1 - y^2)u_{yy} = 0$ at each $(x, y) \in R^2$.

(b) Use the method of separation of variables to find the solution $u(x,t): [0,T] \times [0,L] \rightarrow R$ to the initial/boundary value problem **(10)**

$u_t(x,t) = u_{xx}(x,t)$ $\quad for \quad 0 < t \leq T \; and \; 0 \leq x \leq L,$

$u(x,0) = f(x),$ $\quad for \quad 0 \leq x \leq L,$

$u(0,t) = u(L,t) = 0,$ $\quad for \quad 0 < t \leq T,$

where $f: [0,L] \rightarrow R$ is a known function.

Q. No. 7. **(a)** Use Simpson's 3/8 rule to estimate the integral **(10)**

$$\int_1^3 (x^3 - 2x^2 + 7x - 5)dx.$$

By comparing your answer with exact value, find the error.

(b) Solve the system of equations by Jacobi iterative method. **(10)**

$10x + 3y + z = 19, \quad 3x + 10y + 2z = 29, \quad x + 2y + 10z = 35$

Q. No. 8. **(a)** In the following table values of $y = x + \sin x^2$ are tabulated **(10)**

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$f(x)$	1.84147	2.03562	2.19146	2.29290	2.32521	2.27807	2.14935

Construct a difference table and estimate $f(1.04)$ and $f(1.57)$.

(b) Use trapezoidal and Simpson's 1/3 rules to approximate $\int_0^{\pi/2} sin^2(x)dx$. Find a maximum bound for the error in each case. Compare your approximations with the actual result. **(10)**

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