## APPLIED MATHEMATICS

## TIME ALLOWED: THREE HOURS

MAXIMUM MARKS $=\mathbf{1 0 0}$
NOTE:(i) Attempt ONLY FIVE questions. ALL questions carry EQUAL marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(v) Extra attempt of any question or any part of the attempted question will not be considered.
(vi) Use of Calculator is allowed.
Q. No. 1. (a) Find the directional derivative of $f(x, y, z)=x y^{2}+y z^{2}$ at the point $(2,-1,1)$ in the direction of the vector $i+2 j+2 k$ ?
(b) Evaluate $\int_{c}\left(x y+y^{2}\right) d x+x^{2} d y$ where c is bounded by the line $\boldsymbol{y}=\boldsymbol{x}$ and the curve $\boldsymbol{y}=\boldsymbol{x}^{2}$
Q. No. 2. (a) Find the constants $\mathrm{a}, \mathrm{b}$, and c so that

$$
\begin{equation*}
F=(x+2 y+a z) i+(b x-3 y-z) j+(4 x+c y+2 z) k \tag{10}
\end{equation*}
$$

is irrotational and hence find the function $\psi$ such that $F=\nabla \psi$
(b) The forces $F_{1}, F_{2}, F_{3}, F_{4}, F_{5}$ and $F_{6}$ act along the sides of a regular hexagone taken in order. Verify that all the forces will be in equilibrium if,

$$
\sum \mathrm{F}=0, \text { and } F_{1}-F_{4}=F_{3}-F_{6}=F_{5}-F_{2} .
$$

Q. No. 3. (a) A system of forces acts on a plate in the form of an equilateral triangle of side 2 a . The moment of the forces about the three vertices are $M_{1}, M_{2}$ and $M_{3}$ respectively. Find the magnitudes of the resultant.
(b) If a particle P move with a velocity V given by $V^{2}=n^{2}\left(a x^{2}+2 b x+c\right)$. Show that $P$ executes a simple harmonic motion. Find the centre, the amplitude and the time period of the motion?
Q. No. 4. (a) What is the difference between linear differential equation and Bernoulli's equation? Also find the sotution of the following differential equation.

$$
\left\lfloor x^{x}+y=1-y\right.
$$

(b) Use the method of undetermined coefficient to solve the following differential equation.

$$
y^{\prime \prime}-3 y^{\prime}+2 y=2 x^{3}-9 x^{2}+6 x
$$

Q. No. 5. (a) Solve the equation

$$
\begin{equation*}
0=\frac{1}{2}+\frac{1}{4} x^{2}-x \sin x-\frac{1}{2} \cos 2 x \quad \text { with } x_{0}=\frac{\pi}{2} \tag{10}
\end{equation*}
$$

(b) Derive two point Gaussian integration formula for the following integral and use it to solve the integral.

$$
\begin{equation*}
\int_{1}^{1.6} \frac{2 x}{x^{2}-4} d x \tag{10}
\end{equation*}
$$

Q. No. 6. (a) Determine the second degree polynomials by using Newton's method. Also estimate the value of $f(0.1)$ and $f(0.5)$ for the data.

| $x$ | 0.0 | 0.2 | 0.4 | 0.6 |
| :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 15.0 | 21.0 | 30.0 | 51.0 |

(b) Does the dominate diagonal is necessary for finding the numerical solution of system of linear equations by using Gauss Jacobi's and Gauss Seidal methods. Explain the reason. In what conditions a numerical method is used instead of analytical method? Find the solution of the following system by performing three itrations of Gauss Seidal method.

$$
\begin{aligned}
& 6 x-3 y+z=11 \\
& 2 x+y-8 z=15 \\
& x-7 y+z=10
\end{aligned}
$$

Q. No. 7. (a) Define even function and odd function with examples. Verify that the Fourier

$$
\text { Series for the function } \quad f(x)=\left\{\begin{array}{ll}
0 & \text { When } 0<\mathrm{x}<\pi \\
1 & \text { When } \pi<\mathrm{x}<2 \pi
\end{array}\right\}
$$

is $f(x)=\frac{1}{2}-\frac{2}{\pi}\left(\sin x+\frac{1}{3} \sin 3 x+\frac{1}{5} \sin 5 x\right.$ $\qquad$ .)
(b) Solve the following partial differential equation by using method of separable variable.

$$
\frac{\partial u}{\partial x}=2 \frac{\partial u}{\partial t}+u, \quad \text { given } \quad u(x, o)=6 e^{-3 x}
$$

Q. No. 8. (a) The Trapezoidal rule applied to $\int_{0}^{2} f(x) d x$ gives the value 4 , and the Simpson's rule gives value 2 , what is the value of $f(1)$ ?
(b) Find the first two derivatives at $\mathrm{x}=1.1$ and $\mathrm{x}=1$ from the following data table.

| $x$ | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)$ | 0.000 | 0.1280 | 0.5440 | 1.2960 | 2.4320 | 4.000 |

