FEDERAL PUBLIC SERVICE COMMISSION
COMPETITIVE EXAMINATION-2017
FOR RECRUITMENT TO POSTS IN BS-17
UNDER THE FEDERAL GOVERNMENT

Roll Number

APPLIED MATHEMATICS

TIME ALLOWED: THREE HOURS	MAXIMUM MARKS = 100

NOTE:(i) Attempt **ONLY FIVE** questions. **ALL** questions carry **EQUAL** marks
(ii) All the parts (if any) of each Question must be attempted at one place instead of at different places.
(iii) Candidate must write Q. No. in the Answer Book in accordance with Q. No. in the Q.Paper.
(iv) No Page/Space be left blank between the answers. All the blank pages of Answer Book must be crossed.
(vi) Extra attempt of any question or any part of the attempted question will not be considered.
(v) **Use of Calculator is allowed**.

Q. No. 1. **(a)** Suppose $\mathbf{r}(t) = (e^t \cos t)\hat{i} + (e^t \sin t)\,\hat{\mathbf{j}}$. Show that the angle between $\mathbf{r}$ and $\frac{d^2\mathbf{r}}{dt^2}$ never changes. What is the angle? **(10)**

(b) Using divergence theorem of Gauss, Evaluate $\iint_S x^2 dxdy + x^2 ydzdx + x^2 zdxdy$ where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$, $(0 \le z \le b)$ and the circular disks $z = 0$ and $z = b$, $(x^2 + y^2 \le a^2)$. **(10)**

Q. No. 2. **(a)** A 100 Kg wooden crate rests on a wooden ramp with an adjustable angle of inclination. Draw a free body diagram of the crate. If the angle of the ramp is set to 10^0, find the perpendicular and parallel components of the crate's weight to the ramp. Also find the static friction force between the crate and the ramp. At what angle will the crate just begin to slip? (coefficient of static friction μ_s between wood block and wood surface is 0.28) **(10)**

(b) A ladder having a uniform density and a mass m rests against a frictionless vertical wall at an angle of 60^0. The lower end rests on a flat surface where the coefficient of static friction is 0.40. A person of mass M=2m attempts to climb the ladder. What fraction of the length L of the ladder will the person have reached when the ladder begins to slip? **(10)**

Q. No. 3. **(a)** A particle moving in a straight line starts with a velocity u and has acceleration v^3, where v is the velocity of the particle at time t. Find the velocity and the time as functions of the distance travelled by the particle. **(10)**

(b) A particle describing simple harmonic motion has velocities 5 ft/sec and 4 ft/sec when its distance from the centre are 12 ft and 13 ft respectively. Find the time-period of motion. **(10)**

Q. No. 4. **(a)** Check for the exactness and solve the following ordinary differential equation. **(10)**
$(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + \sec^2 y)dy = 0$

(b) Solve the following second order differential equation: **(10)**
$$\frac{d^2y}{dx^2} + 4x = \sec 2x$$

Q. No. 5. **(a)** Solve the initial value problem. **(10)**
$$\frac{d^2y}{dx^2} = 2\text{-}6x,\ y'(0) = 4,\ y(0) = 1$$

(b) Solve the following Boundary Value Problem. **(10)**
$y' + 4y = 0$, $y(0) = -2$, $y(2\pi) = -2$
How many solutions do you get for this problem?

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Q. No. 6. **(a)** Compute the Fourier series for the function x^2 on the interval $0<x<L$, using as a basis of function with boundary conditions $u'(0) = 0$ and $u'(L) = 0$. Sketch the partial sums of the series for 1, 2, 3 terms. **(10)**

(b) Find a solution to the following partial differential equation that will also satisfy the boundary conditions. **(10)**

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}, \quad u(x,0) = f(x),\ u(0,t) = 0,\ u(L,t) = 0$$

Q. No. 7. **(a)** Use bisection and false position methods to locate the root of $f(x) = x^{10} - 1$, between 0 and 1.3. Which of the two methods is better and why? **(10)**

(b) Use Lagrange interpolation polynomial of the first and second order to evaluate f(x) at x=15 for the following data; **(10)**

x:	0	20	40
f(x):	3.850	0.800	0.212

Q. No. 8. **(a)** Use Trapezoidal Rule with 3 segments to evaluate. **(10)**

$$\int_0^{0.8} x \cdot e^{2 \cdot x}\,dx$$

Also find the true solution and the percentage error.

(b) Consider the function $f(x) = xe^x$. Obtain approximations to $f'(2)$ with $h = 0.5$ using forward, backward and central difference formulas. **(10)**

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